The Tyranny of Chronological Age

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Abstract

This paper presents an analysis of a phenomenon known as "The Relative Age" effect. When assessing the innate ability (or talent) of individual children who are grouped into age cohorts, systematic errors occur due to differences in biological maturity. A structural model of a multi-period progression through levels (or grades) that employs screening and selection is developed. Through a series of simulations, impact of the relative age on the of selection process is analyzed.

1. INTRODUCTION

A review of player profiles the National Hockey League (NHL) reveals that more than four times as many players are born in January as in December. Further, 70 percent of all players are born in the first six months of the calendar year. Investigations into the highest level of amateur hockey leagues reveal similar findings. The obvious question is why do players born early in the calendar year dominate these leagues? When we look back to the years that these players were born, we find that the monthly birth rate of males does not conform to this pattern. For the years that the players in question were born, the birth rate of males in Canada is almost perfectly uniform, with no evidence of seasonality or other systematic variations. Researchers in the fields of education and psychology have labeled this observed phenomenon as the *Relative Age Effect*.

Players who end up in the NHL (or in the upper levels of Canadian Hockey) are picked or selected by a non-price allocation mechanism, which makes this issue of interest to economists. The allocation mechanism is based on relative performance – children who do well tend to stay in the system and those who demonstrate an exceptional aptitude are given special treatment. Those who excel are placed on teams that receive dedicated resources. This type of selection mechanism is expected to filter out untalented players, to encourage talented players to stay in the system and to develop their talents. Yet observed results, summed up by what has been identified as the relative age effect, strongly suggests otherwise, raising the question as to why is this apparently sensible allocation mechanism producing such birth-date biased results?

The phenomenon described above is not unique to hockey. Similar results are found in most organized sports played in many countries and cultures[2]. Further,

[2] A study of teams in the under 23 year old world cup of soccer found that there was strong evidence of the relative age effect across all teams. This event draws on teams from 24 countries.

studies of academic performance in school systems of different countries has also shown a bias similar to that found in hockey. In national exams given to elementary and high-school students the top percentiles tend to be dominated by children and youths who are born in the first half of the calendar year[3]. In contrast, grade school retention's (failures) and even teenage suicides (see figure 1) tend to be dominated by children born in the last half of the calendar year[4].

Recent research in Psychology has shown that selection processes in schools and some sports are characterized by systematic errors caused by the difficulty of observing ability independent of maturity in children[5]. When children are grouped into age categories such as calendar year of birth, there may be differences in both physical and mental maturity due to the differences in age within the group. If children are grouped by calendar year, the age difference between any two children can be as much as twelve months. During the formative years, this may produce significant differences in performance due to the difference in maturity and ability of these children. When assessing the performance of children during this time, there is a risk of mistaking differences in innate ability with differences in maturity.

Furthermore, in many institutions, such as minor hockey, children are often sorted into tiers based on observed ability. Within these tiers children will receive different levels of training. Usually those demonstrating the highest ability will be sorted into a tier which will receive a greater degree of training than those who demonstrated a lesser ability. If the relative age effect is present during this sorting process, then children of greater maturity will be mistakenly selected into the highest tiers. Once selected, the differences in training will cause the selection bias to persist beyond the point in time where relative age effects are significant. This secondary effect is referred to as the *Training Effect*. It is generally recognized that the relative age effect is transitory in nature, disappearing by early teenage years. However, if it occurs concurrent with the training effect, temporary distortions observed in the data cited will become permanent.

Barnsley and Thompson (1988) propose two hypotheses to explain the observed age distribution found in professional hockey and the highest levels of amateur hockey. The first suggests that there is a tendency for older children to continue to participate in hockey over time, whereas those who are younger tend to drop out. This is referred to as the *discouragement effect*. The second hypothesis suggests that the older children experience greater success due to being streamed into tiers, where the top tiers receive more intensive training. The better training causes them to continue to progress through the system in the subsequent periods and it

[3]See: Freyman, R. "Further Evidence on the Effect of Date of Birth on Subsequent School Performance." *Educational Research,* 8, 58-64, (1965); Jinks, P.C., "An Investigation into the Effect of Date of Birth on Subsequent School Performance." *Educational Research,* 6, 220-225, (1964); Russell, R.J.H. and Startup, M.J. "Month of Birth and Academic Achievement." *Personality and Individual Differences,* 7, 839-846, (1986); Sutton, P. "Correlation between Streaming and Season of Birth in Secondary Schools." *British Journal of Educational Psychology,* 37, 300-304, (1967); Thompson, D. "Season of Birth and Success in the Secondary School." *Educational Research,* 14, 56-60, (1971).

[4]See: Barnsley, R.H., Allen, J., and Thompson, A.H. "School Achievement, Grade Retention and 'The Relative Age Effect',"

[5]Barnsley, R.H., Thompson, A.H. and Barnsley, P.E.,"Hockey Success and Birthdate: The Relative Age effect."*Canadian Association for Health, Physical Education, and Recreation,* 51, 23-28,(1985);

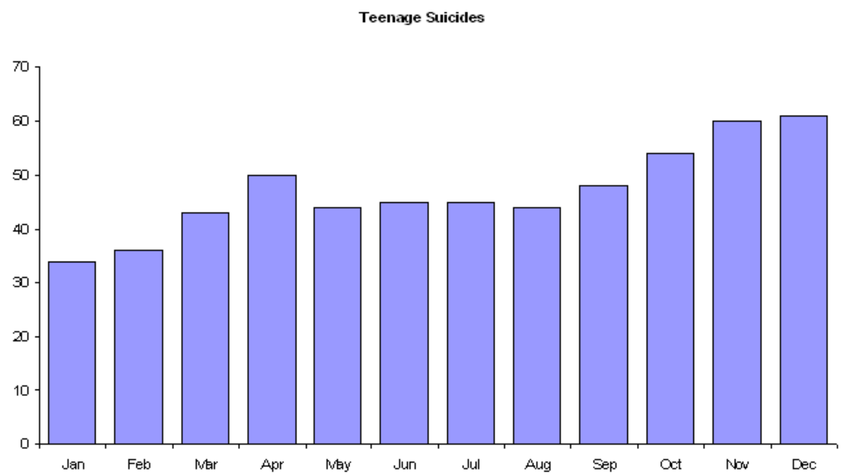


FIG. 1 Teenage suicides by month of birth, 1979-1990 *(Source: Alberta Ministry of Health)*

is this top tier that supplies players to the professional leagues. The work of Anders Ericsson and others[6] strongly suggest that talent alone is never enough to explain success at the elite levels of sports. While talent may be considered a precondition, intensive training is a necessary condition for success.

Whether it is the selection of elite teams in minor hockey or allocating young students into enriched classes in primary school, an allocation of resources is taking place, thus making this inherently an economic problem. By allocating resources to those who have the potential to derive the most benefit, there is an implicit social gain. In the case of education, society, by streaming, hopes to produce the best doctors, researchers and teachers. If there is a limit to the number of doctors a society can produce, then the hope is to select the best candidates to fill those positions. If there exists some systematic bias the distorts the signal used in the streaming process, this creates a potential cost to society that would reduce social welfare.

The social cost of the relative age effect can be extensive. It can range from the lost opportunity of not training the best team of cancer researchers, to the associated costs of health care and crime due to those children who have been disenfranchised by society. In addition to the problem of teenage suicide, recent studies have linked problems of self-esteem and mental health to the relative age effect. The existence of the relative age effect implies that there is a potential for two types of measurement errors. First, relatively older children will be deemed to possess higher natural ability than is the case, placing pressure on these individuals to maintain a level of performance that exceeds their true ability. Second, relatively

[6] Ericsson, K. A., Charness, N., Feltovich, P. J., Hoffman , R. R., (editors) **The Cambridge Handbook of Expertise and Expert Performance** (June 2006) Cambridge Press

younger children will be overlooked in the process due to lack of maturity. This may cause them to become discouraged and withdraw from the process prematurely. Both of these errors lead to an inefficient use of society's resources and potential hardship for the individuals involved.

The goal of this essay is to develop a structural model that captures the various factors that influence the observed age distribution found in organized hockey. Through the use of a structural model we hope to separate differences in innate ability from differences in maturity and training. In doing this, we can estimate the magnitude of the misallocation of players due to the relative age effect. It is hoped that this work will lead to a framework that can address the more general issues that surround all education and training processes that involve streaming.

The analysis is applied to Canadian minor hockey for two reasons. First, the CAHA is a fairly rigid system that is applied to all Canadian hockey players on a national level. The CAHA has very strict rules governing eligibility based on birthdate and geographical location. The level of hockey a Canadian youth plays is completely determined by birth date. Since the CAHA is a national body, any relocation of a hockey player will have no bearing on the player's level. Second, minor hockey tends to mirror the school system with respect to age levels, cohort groupings, entry dates and end dates. Both systems use some form of "streaming" to allocate scarce resources. Hockey has "rep-teams" for the top hockey players and schools often have enriched classes for the top students. Finally, since hockey is an elective activity, players may choose to no longer participate; which is not true in the education system; therefore hockey supplies a much cleaner set of data for analysis.

2. THE CANADIAN MINOR HOCKEY SYSTEM

Historically, 80 percent of the players in the NHL are Canadian citizens and a product of Canadian minor hockey system. This system is governed by the Canadian Amateur Hockey Association (CAHA), a long-standing organization which has governed and regulated amateur hockey in Canada for several decades. In this role, the CAHA has kept accurate and detailed records on all aspects of minor hockey, including team and individual statistics, player histories and demographics. Canadian NHL'ers typically entered minor hockey when they were between 6 and 8 years of age. Each subsequent year they would progress through a series of levels, or leagues, until they ultimately reached the highest amateur level, known as "Junior". It is from this level that players are selected for the NHL.

As we mentioned earlier, a large majority of the players in Major Junior hockey (17-19 year old's) are born in the first six months of the calendar year. The top box in figure 2 presents a summary of birth data collected from the 1983 rosters of the Western Hockey League. We see that 71.5% of the players are born between January and June. Further, when the players are grouped by quarters, players born in the first three months of the year account for 41.9 % of the players followed by a systematic decline in participation to only 9.2% by players born in the last three months of the year.

Month of Birth
Western Hockey League
1983 Protected List (Total = 698)

January	**15.5%**		
February	**14.5%**	**41.9%**	
March	**11.9%**		**71.5%**
April	**11.0%**		
May	**10.9%**	**29.6%**	
June	**7.7%**		
July	**7.7%**		
August	**6.0%**	**19.3%**	
September	**5.6%**		**28.8%**
October	**3.6%**		
November	**2.4%**	**9.2%**	
December	**3.2%**		

Source: Barnsley, Thompson, and Barnsley (1985)

Percentage of Live Male Births by Month in Canada,
July 1, 1966 to June 30, 1967

Canadian Statistical Review, 1968

January	9.0%	July	8.7%
February	7.4%	August	8.7%
March	8.0%	September	8.7%
April	8.2%	October	8.3%
May	9.2%	November	7.9%
June	8.2%	December	7.7%

FIG. 2 Comparison of the participation rate in the Western Hockey League (18 year olds) and the corresponding national birthrate of 18 year olds in Canada

How does this result compare to the total population of males at that age? Since the average age of the Western Hockey League is 17 years, we look back at the approximate time frame that most players would have been born. This corresponds to the years 1966-1967. The lower box in figure 2 shows the number of live male births by month for the period of July 1, 1966 to June 30, 1967. We can see that the distribution of male births by month is fairly uniform, and not at all similar to the distribution found in the Western Hockey League.

Records from the CAHA show the participation rate by month of birth at the youngest level of minor hockey very much mirrors that of the male birth rate in Canada. In other words, children grouped by month of birth tend to be uniformly represented at the entry level of minor hockey. Assuming that the distribution of natural ability is independent of the month of birth, we would expect that players at the highest level of both amateur and professional would also be uniformly distributed by month of birth. As mentioned above, this is not the case.

At this point we shall describe in more detail the process by which a player who enters minor hockey at the age of six years may progress through the system until an age of between16 and 18 years. Figure 3 is a summary of the number of players at each level found in a representative minor hockey system[7]. At each level players are grouped into two cohorts. Those born in the first six months of the calendar year are designated as *Old* and those born in the last six months are designated as *Young*. Also included is the percentage of the total age group each cohort represents.

At the first stage, known as "Mites", players are randomly allocated into teams in order to play recreational hockey. At this level the emphasis is on skill development and recreation rather than competition. However, there is also an evaluation process being carried out at this time which will influence the path each player will follow in subsequent years of playing minor hockey.

After playing one or two years at the Mite level, players move to the next level of organized hockey ("Pups"). At the next level there are two possible categories, or states, a player may find himself : tier one (*Rep-Team*), or tier two (*Recreational Hockey*). In addition players may also choose to leave minor hockey altogether (*Exit*). Tier one is a selective group chosen from the list of players in stage one who were evaluated to be the best hockey players in the previous level. The players who are determined to be the best are offered the opportunity to play on an elite team which offers them more ice time, better coaching, and a higher level of competition.

Players not selected for tier one, or those selected but declined the offer to play tier one, can then choose to play recreational- or tier two- hockey . This category, or state, involves less ice time and usually a lower level of competition. Tier two is open to any individual wishing to play, with the only restriction being that they must play with others who are born in the same calendar year. Finally, some players may choose to enter state three and stop playing minor hockey.

The selection process described above repeats each year until players reach an age of 16 to 18 years. At the end of each hockey season players in both tier one

[7] This data set is from the Edmonton, Alberta chapter of the CAHA system.

Level	Tier	Old	Young	total	Total all tiers	Exit
1 Mite	n/a	1132	1078	2210	2210	
8 & under		51%	49%	100%		
2 Pup	Tier 2	760	812	1572		
9-10 yr		34%	37%	71%		
	Tier 1	216	106	322	1894	316
		10%	5%	15%	86%	
3 Peewee	Tier 2	602	623	1225		
11-12 yr		32%	33%	65%		
	Tier 1	213	116	329	1554	340
		11%	6%	17%	82%	
4 Bantum	Tier 2	475	488	963		
13-14 yr		31%	31%	62%		
	Tier 1	148	73	221	1184	370
		10%	5%	14%	76%	
5 Midget	Tier 2	314	297	611		
15-16 yr		27%	25%	52%		
	Tier 1	81	44	125	736	448
		7%	4%	11%	62%	

FIG. 3 Summary data of the minor hockey system (CAHA Edmonton, Alberta 1984). Participants born in the months January to June are grouped as "Old", participants born in the months July to December are grouped as "Young"

and tier two are re-evaluated and a new list made of players to be offered positions in tier one. Those players not offered a position on a tier one team again have the choice of continuing in tier two or leaving the sport. The flow chart found in figure 4 illustrates the entire process. The transition arrows between stage two and stage three of figure 4 illustrate the possible paths that players may follow each year. It is the transition from stage two to stage three that describes the process by which players move through the several levels of organized minor hockey, with only difference found in the stage one to stage two transition, the initial sorting process. Note that a player in any given state during a hockey season could move to any of the three possible states the next season.

The process continues until the final period when players have reached the highest league governed by the CAHA (usually known as *"Midget"*). After this level, players cease to be governed by the CAHA. The next level available to players is the Canadian Hockey League (CHL). The CHL is made up of three leagues: the western hockey league (WHL); the Ontario hockey league (OHL); and the Quebec major junior hockey league (QMJHL). These three leagues are the highest amateur leagues in Canada. The CHL selects players from the CAHA's midget level through a combination of regional protected lists and an annual draft. The CHL is strictly an elite league which produces players for the professional draft.

Players from midget hockey who are not selected by a team in the CHL may have the option of playing tier two junior hockey. Tier two junior is a provincial league that is of slightly lower calibre than the CHL. However, Tier two junior is also a competitive league that takes a limited number of players in a similar manner

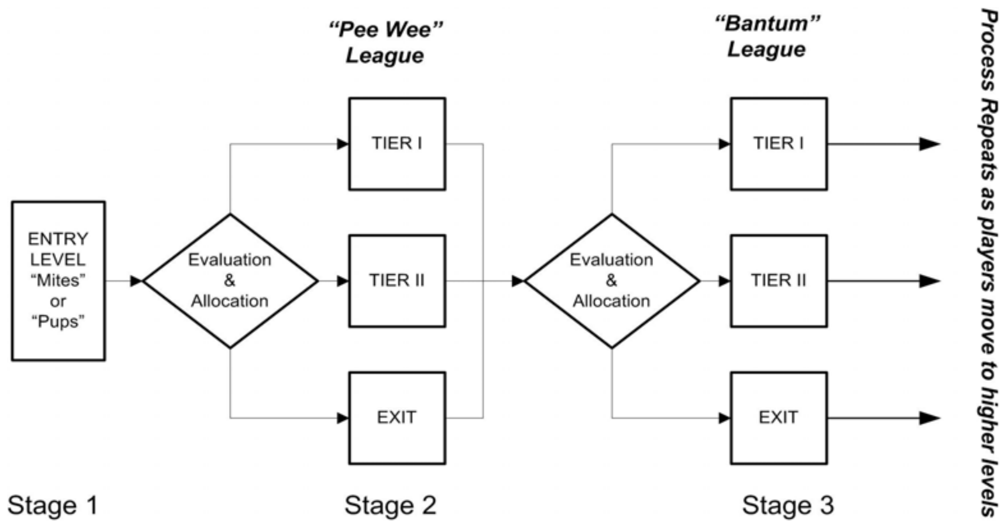


FIG. 4 Minor hockey evaluation and progression flowchart

to the CHL. Players who fail to make either a CHL team or tier two have very few options to continue in organized, competitive hockey. Some may join a recreational men's league and some may try to play for a college or university (contingent on academic requirements). However, for most, their hockey careers end at this stage.

Comparing the participation rate by age cohort at the Mite level to the age cohorts at the Midget level in figure 3 demonstrates the relative age effect. Figure 4 shows us all the possible paths that an individual may follow to reach the final stage of the process. Obviously some paths are much more likely, while others are highly improbable. For example, it is conceivable that an 18 year old that has never played organized hockey may show up to a junior tryout camp and make the team, such an event is almost unheard of in modern hockey. Therefore, not all the paths described in figure 4 can be considered relevant. This allows us to apply some simplifying assumptions to make the understand of the process more tractable. The next section presents an overview of the model where we introduce the assumptions and formulate our hypothesis.

3. THE MODEL

A structural model is a framework that imposes a series of simplifying assumptions to a problem that makes the process tractable. The objective is to capture the essential features of the problem as a system of equations. The various factors influencing the model are treated as exogenous variables in the system of equations. The outcomes, or predictions, of the model are the endogenous variables whose values are the result of solving the system of equations.

The progression through the minor hockey system can be described by a *Markov* process. At any point in time a player will be in one of three states: tier one, tier

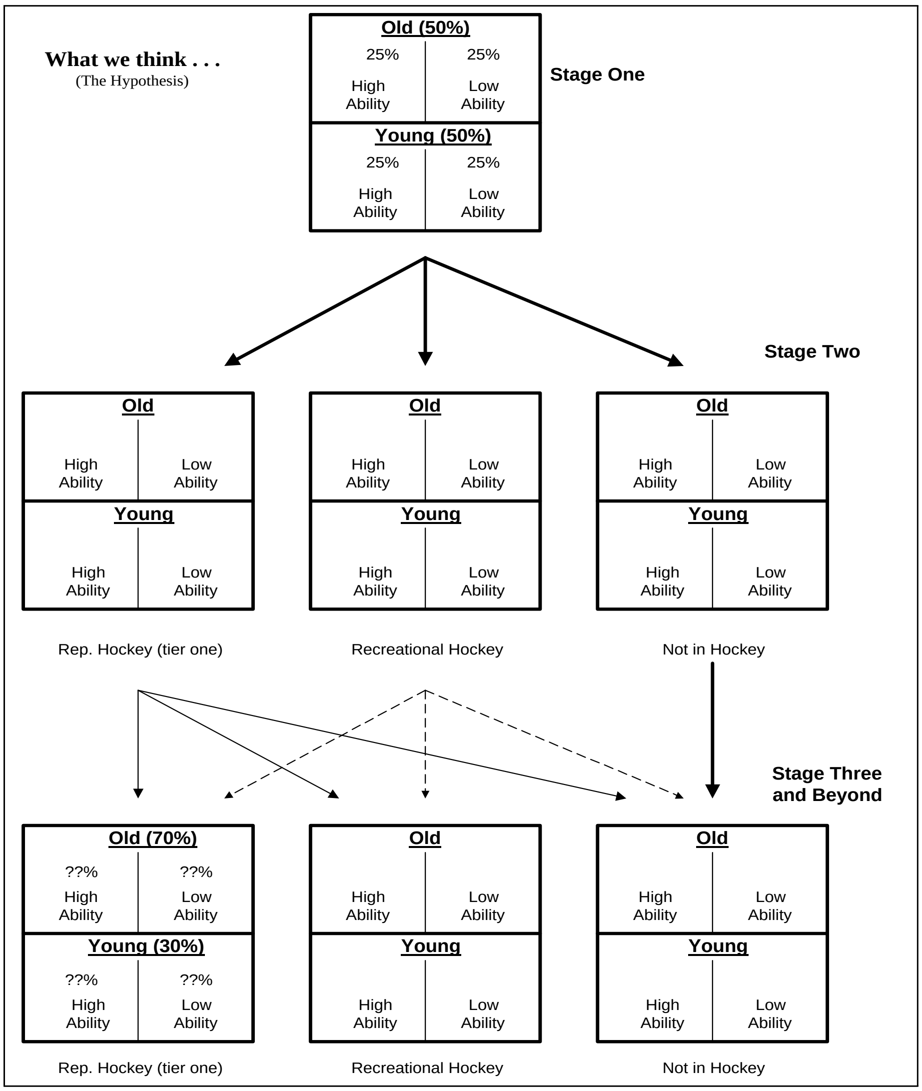


FIG. 5 Outline of the structural model of hockey

two or exit. The player will then move to one of the same three states in the next period (year) based on a set of transitional probabilities. The probabilities are determined by the characteristics of hockey players. The characteristics that identify a player are natural ability, relative age, and level of training. These three characteristics completely describe a player.

By determining the set of probabilities at each stage, we can produce a dynamic process that replicates what is observed in minor hockey. Through the use of the computer and maximum likelihood techniques, the exogenous variables can be adjusted, or calibrated, such that the model produces a process similar to what is observed in the data. It is through such an estimation that the influence of natural ability, which cannot be directly observed, can be inferred and separated from those influences which are directly observable.

3.1. Assumptions

The first assumption of the model is that natural ability is independent of month or season of birth. Therefore the distribution of natural ability will be the same in each grouping of players based on birthdate. Furthermore, we assume that players are either of high ability or low ability and that there are equal numbers of both in the population. Thus any initial grouping will have 50% high ability and 50% low ability players. Second, we classify players within a calendar year as either young or old. Since both groups are born in the same calendar year, the difference between the two are what is called relative age. We assume that initially there are equal numbers of both age groups.

Based on our first two assumptions we have two characteristics that completely describe the population of players who enter the hockey system: relative age and natural ability. This implies four distinct groupings of our initial population. First they can be divided into either young and old players. then each of these two groups can be further divided into high ability or low ability. Since we assumed equal numbers for each characteristic, then each of the four groupings will have equal numbers of players. The four groups are illustrated in the stage one box found in figure 5.

The third assumption of the model is that, all else held constant, players with high natural ability are more likely to move to tier one than players of low natural ability. Further, high ability players are also more likely to move to tier two and are less likely to exit than low ability players. Therefore, accounting for all other factors, we would expect tier one to be dominated by high ability players and the exit state to be dominated by low ability players. It is assumed that the influence of natural ability is unchanged throughout the entire process. We allow for some " noise", or randomness such that there will be both types of players found in all three states. The degree of noise in the model is determined from the maximum likelihood estimations.

The fourth assumption is that relative age effects the probability of a player being selected for tier one. It is assumed that old players are more likely to move to tier one than young players. Further, high ability players are also more likely to move to tier two and are less likely to exit than low ability players. Therefore, accounting for all other factors, we would expect tier one to be dominated by high ability players and the exit state to be dominated by low ability players.

The fifth assumption of the model is that players who are in tier one in the current period are the most likely to move to tier one in the next period than players who are either in tier two or out of the system. This is the *training effect*, which captures the influence of players receiving better coaching and greater ice time. The training effect is not considered to be cumulative. only the state that the player is currently in matters; not the player's overall history. In fact we make the extreme assumption that once a player is out of hockey never re-enters either tier one or Tier Two. The assumption that players initially in tier one has the highest probability of moving to tier one in the next stage captures the *Training Effect*.

Finally, the relative age effect, is assumed to decay at each stage of the process, such that it is strongest in the stage one to stage two transition and weakest in the last transition period. The training effect is assumed to persist throughout the process.

The multi-staged process described above is known as a *Finite Markov Chain*. The transition probabilities at each step of the process make up what is referred to as the Markov *Transition Matrix*. The next section develops a formal model of the Markov process which will allow for a detailed statistical analysis of the minor hockey system.

3.2. Formalizing The Model

3.2.1. Initial Conditions

Each year a group of children born in the same calendar year begin playing hockey for the first time. The league first time players enter is named *peewee pup*. These players are randomly placed on teams where they all receive the same training and playing time. We can categorize the players in this league by two characteristics: relative age and natural ability. Any player who is born in the first six months of the calendar year as *old* and those born in the last six months of the year as *young*. Within each grouping (young or old), any given child is either a high ability player or a low ability player. We assume that half the children will be of high ability and half of low ability. These two characteristics allows us to categorize these children into four groups:

Group I:	old with high ability	**(OH)**
Group II:	old with low ability	**(OL)**
Group III:	young with high ability	**(YH)**
Group IV:	young with low ability	**(YL)**

At the beginning of the second year, children in each group will find themselves in one of three states: either level A (tier one), level B (tier two), or they exit hockey altogether (E). Let A^i, B^i, and E^i denote the probabilities of any given child from group i ($i = I, ..., IV$) moving to level A, level B, and Exit respectively.

For example, consider group I moving from year one to year two. We can write the process as

$$OH_{t=1} \times \begin{bmatrix} A^I & B^I & E^I \end{bmatrix} = \begin{bmatrix} oh_A & oh_B & oh_E \end{bmatrix} = OH_{t=2}$$

where OH_1 is the number children in group I in year one, $\begin{bmatrix} A^I & B^I & E^I \end{bmatrix}$ is the vector of transition probabilities, and $\begin{bmatrix} oh_A & oh_B & oh_E \end{bmatrix}$ is a vector containing the number of group I children in each of the three possible states in year two; this vector is denoted by OH_2. In a similar fashion we can describe the transition process for groups II, III, and IV as follows:

$$\begin{array}{lll} OL_1 \times \begin{bmatrix} A^{II} & B^{II} & E^{II} \end{bmatrix} & = \begin{bmatrix} ol_A & ol_B & ol_E \end{bmatrix} & = OL_2 \\ YH_1 \times \begin{bmatrix} A^{III} & B^{III} & E^{III} \end{bmatrix} & = \begin{bmatrix} yh_A & yh_B & yh_E \end{bmatrix} & = YH_2 \\ YL_1 \times \begin{bmatrix} A^{IV} & B^{IV} & E^{IV} \end{bmatrix} & = \begin{bmatrix} yl_A & yl_B & yl_E \end{bmatrix} & = YL_2 \end{array}$$

In general, OH_t will denote the vector of states of group I children in year t. Similarly, OL_t, YH_t, YL_t will denote state vectors at time t for children from groups II, III, and IV, respectively.

From the above process we see that all four groups of children are distributed across the same three states in year two. As we move from year two to year three, a child from any group found in any given state in year two can move to any given state in year three. The possible states in year three are the same as year two. For each subsequent year the possible states will be the same as the previous year's states. Starting in year two, the markov transition matrix for each of the four groups can be written as:

$$[M_t^i] = \begin{bmatrix} AA_t^i & AB_t^i & AE_t^i \\ BA_t^i & BB_t^i & BE_t^i \\ 0 & 0 & 1 \end{bmatrix} \qquad \left\{ \begin{array}{l} t = 2, ..., n \\ i = I, II, III, IV \end{array} \right\} \tag{1}$$

where each element in the matrix represents the probability of moving from a given state at time t to any given state at time $t+1$. For example:

- AA_t^i is the probability of a group i child in state A at time t moving to state A at time $t+1$;
- BA_t^i is the probability of a group i child in state B at time t moving to state A at time $t+1$;
- AB_t^i is the probability of a group i child in state A at time t moving to state B at time $t+1$;
- and so on...

Note that the third row comprises the vector $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$. This reflects the fact that once a child has exited hockey his probability of entering Level A or level B is zero, thereby making the probability of him staying out of hockey exactly one. Therefore the process is an *absorbing markov chain.*

The distribution of any group of children across states in year n can be expressed as the original vector in year two multiplied by the product of the transition matrices from year two to year n. Using our example of group I children (OH), the final distribution in year n can be expressed as follows:

$$OH_n = OH_2(M_2^I)(M_3^I)...(M_{n-1}^I)(M_n^I) \quad (2)$$

For each of the four groups there is an equation much like equation 2 that describes each group's progression through the minor hockey ranks[8]. In any given year the probabilities in each group's markov transition matrix may differ from those of the other groups'. In other words, the probability of a player in a given state (A or B) at time t moving to a particular state in year $t+1$ is a function of which group he belongs to. The reasons for these differences will be developed in the next section.

3.2.2. The Logistic Model

So far, we have specified that the probability of success in hockey is a function of three variables: natural talent, the level of training, and the relative age of the player. Now we turn to the problem of adding structure to the model by specifying the functional form of the probabilities in the markov process. For this we have chosen the *logistic model,* commonly known in econometrics as the *Logit model*[9]*.*

The first period probabilities are characterized as

$$\begin{aligned} A^i &= \frac{e^{Z_A^i}}{1+e^{Z_A^i}+e^{Z_B^i}} \\ B^i &= \frac{e^{Z_B^i}}{1+e^{Z_A^i}+e^{Z_B^i}} \\ E^i &= \frac{1}{1+e^{Z_A^i}+e^{Z_B^i}} \end{aligned}$$

where

$$\begin{aligned} Z_A^i &= \alpha_0^1 + \alpha_1^1 AGE + \alpha_2^1 TALENT \\ Z_B^i &= \beta_0^1 + \beta_1^1 AGE + \beta_2^1 TALENT \end{aligned}$$

In the above specification, the $\alpha' s$ and $\beta' s$ are coefficients to be determined. AGE is a binary variable which takes on a value of 1 if the player is old and 0 if the player is young. $TALENT$ is also a binary variable which takes on a value of of 1 if the player has talent (high natural ability) and 0 if the player has no talent (low natural ability).

For periods *2* through n, the probabilities in the markov transition matrix $\left([M_t^i]\right)$ specification

[8] If the markov transition matrix was a regular markov chain, where the probabilities were exogenous and constant over time, the previous expression would simply collapse to

$$OH_n = OH_2\left[M_2^I\right]^n$$

[9] The logistic model has long been popular in qualitative analysis for a couple of reasons. First, it's simplicity allows researchers to economize on the number of variables to be estimated– frequently a serious issue in empirical work– as well, it's form makes it manageable to manipulate, allowing for closed form solutions. Second, the logistic model closely approximates a normal cumulative distribution function.

$$\begin{bmatrix} AA_t^i & AB_t^i & AE_t^i \\ BA_t^i & BB_t^i & BE_t^i \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{e^{Z_{AA}^{it}}}{1+e^{Z_{AA}^{it}}+e^{Z_{AB}^{it}}} & \frac{e^{Z_{AB}^{it}}}{1+e^{Z_{AA}^{it}}+e^{Z_{AB}^{it}}} & \frac{1}{1+e^{Z_{AA}^{it}}+e^{Z_{AB}^{it}}} \\ \frac{e^{Z_{BA}^{it}}}{1+e^{Z_{BA}^{it}}+e^{Z_{BB}^{it}}} & \frac{e^{Z_{BB}^{it}}}{1+e^{Z_{BA}^{it}}+e^{Z_{BB}^{it}}} & \frac{1}{1+e^{Z_{BA}^{it}}+e^{Z_{BB}^{it}}} \\ 0 & 0 & 1 \end{bmatrix}$$

where

$$\begin{aligned} Z_{AA}^{it} &= \alpha_0^t + \alpha_1^t AGE + \alpha_2^t TALENT + \alpha_3^t TRAINING \\ Z_{AB}^{it} &= \beta_0^t + \beta_1^t AGE + \beta_2^t TALENT + \beta_3^t TRAINING \\ Z_{BA}^{it} &= \alpha_0^t + \alpha_1^t AGE + \alpha_2^t TALENT \\ Z_{BB}^{it} &= \beta_0^t + \beta_1^t AGE + \beta_2^t TALENT \end{aligned}$$

Since the relative age effect is assumed to diminish as children reach maturity, a decay variable was included in the model. The decay variable entered the model through α_1, β_1, the coefficients on the binary variable, AGE. The specific form of the decay process was

$$\alpha_1^t = \alpha_1^0 e^{-rt} \quad and \quad \beta_1^t = \beta_1^0 e^{-rt} \tag{3}$$

where r is the rate of decay, t is time measured in periods of the process and α_1^0, β_1^0 is the initial magnitudes of the relative age effect at time $t = 0$.

4. SIMULATIONS OF THE MODEL

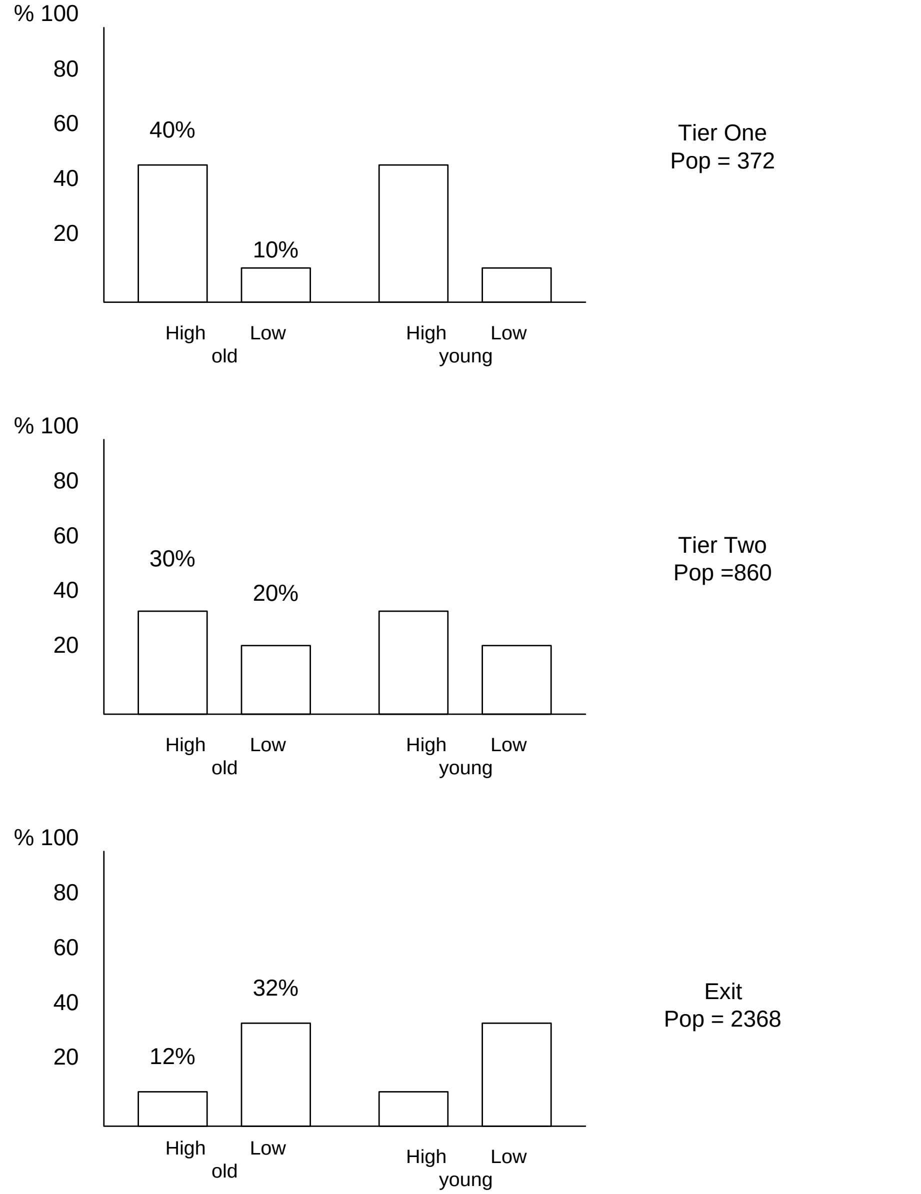
4.1. Initial Conditions

This section presents the results of simulations run on the model. An algorithm was written in *Maple* which carried out a multi-period markov process. Inputted into the program was the relative age variable, the training effect variable, the natural ability parameter, and the population in each category. In the simulations the total population was set at 3600. The population was further divided into four equal groups of 900, based on whether they were young or old, and were of high or low ability. The number of periods was set at five, consistent with the number of levels found in most Canadian minor hockey systems.

The model was then calibrated to produce a final period aggregate distributions across the three states (tier one, tier two, exit) that was consistent with observed populations found in the empirical data[10]. The calibration was carried out with the relative age and training effects set equal to zero and applying restrictions to the size of each state at every period. The value of the natural ability coefficient was estimated from the data for the first stage transition using maximum likelihood. A more detailed discussion of the calibration is found in Appendix I.

[10] Barnsley and Thompson (1988)

FIG. 6 Simulation #1



Simulation Result #1 | Rel. Age = 0% | Training = 0% | Talent = 10%
Decay = 0 | periods = 5

4.2. Simulation Results for a variety of Relative Age and Training effects

4.2.1. Simulation 1: calibration

The top graph in figure 6 shows the distribution of individuals found in tier one after 5 periods. Of the 3600 individuals in the simulation, 372 reached tier one by the fifth period. The 372 were equally divided into young and old players, since the relative age effect in this case was zero. Furthermore, 80% of the players were of high ability and 10% were of low ability.

The middle and bottom graphs illustrate the results for tier two and exit, respectively. By the fifth period there were 860 individuals in tier two and 2368 individuals had exited the system. Again the distribution between young and old in each state is symmetric due to the zero relative age effect.

Once the model was calibrated, a series of simulations were carried out varying the both the magnitude and the rate of the decay of the relative age effect. Initially the simulations were carried out with the training effect set equal to zero, then they were repeated while varying the level of the training effect. A sample of the results are presented in figures three through six.

4.2.2. Simulation 2: Relative Age = 20%, Decay = 10%, Training = 0%

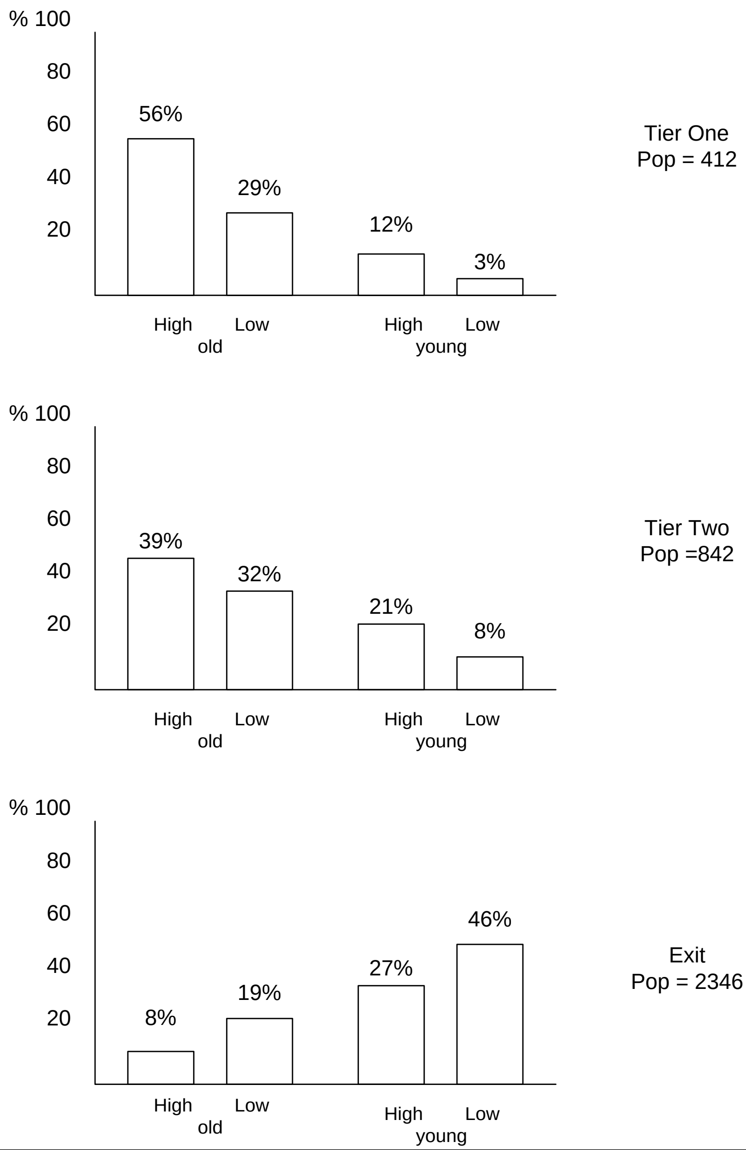
In figure 7 the magnitude of the relative age effect was set at 20%[11] while the rate of decay was set at 10%. After five periods, the results are quite pronounced. In tier one, 56% of the individuals are of the older, high ability category, while only 12% are from the younger, high ability category. Furthermore, 29% of tier one is made up of older, low ability players and only 3% younger, low ability players. In comparing the results to figure 6, we see that older, high ability player participation increased by 16% in tier one, where younger, high ability player participation dropped by 28%. The participation by older, low ability players increased by 19%. In total, older players make up 85% of tier one participation after five periods.

Similar results are found in tier two after five periods. Older individuals represented 71% of tier two, whereas only 29% were from the younger categories. The older, high ability individuals increased their participation in tier two by 9% and older, low ability individuals increased their participation by 12%. younger, high ability individuals participation dropped by 9% and younger, low ability individual dropped by 12%.

Examining the exit state in figure 7, we see that 73% of the individuals who exit the system by the fifth period are from the younger categories; 27% of the high ability type and 46% of the low ability type. Only 8% of the older, high ability individuals have exited the system and 19% of the older, low ability individuals have exited. In comparison to figure 6, older, high ability individuals went from 12% to 8% of the number who exited whereas the younger, high ability individuals who exited went from 12% to 27% of the total.

[11] A 20% relative age effect in this context means that, *ceteris paribus*, an individual born in the first half of the year has a 20% greater chance of being selected for tier one as compared to an identical individual born in the last half of the year in the first period.

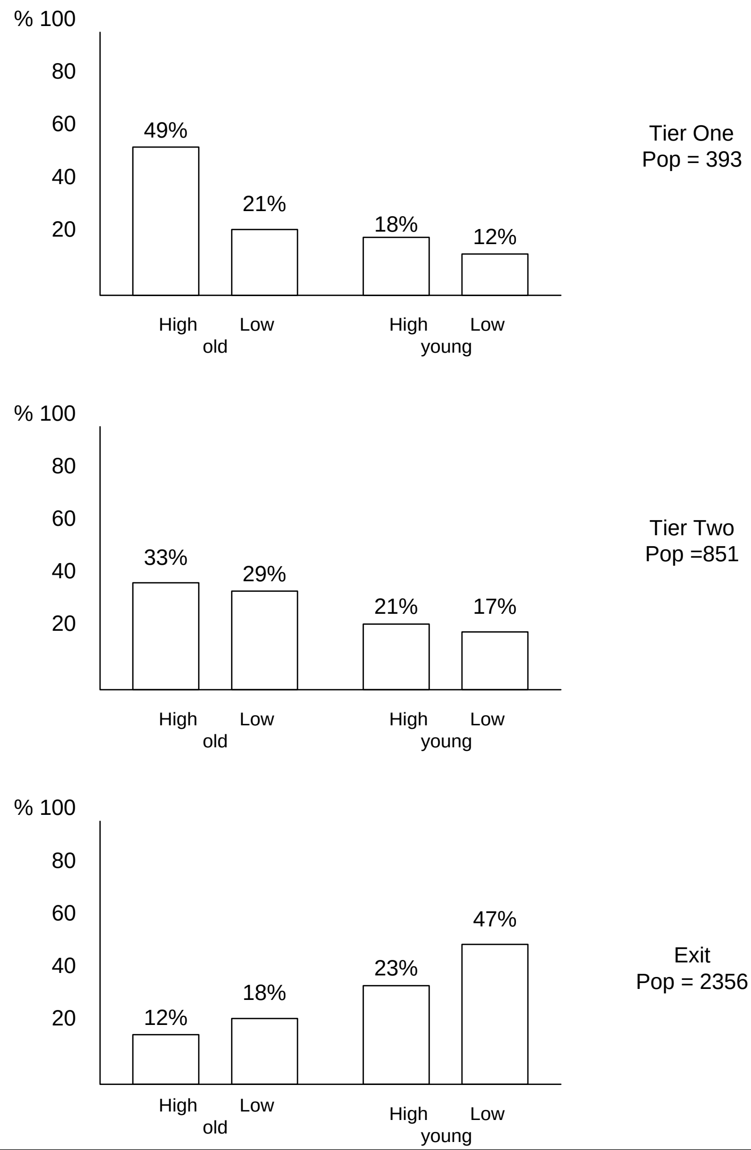
FIG. 7 Simulation #2



Simulation Result #2 Rel. Age = 20% Training = 0% Talent = 10%
Decay = 10%/period periods = 5

Initial conditions: population: 3600 OH = OL = YH = YL = 900

FIG. 8 Simulation #3



Simulation Result #3 Rel. Age = 20% Training = 0% Talent = 10%
Decay = 30%/period periods = 5

Initial conditions: population: 3600 OH = OL = YH = YL = 900

4.2.3. Simulation 3: Relative Age = 20%, Decay = 30%, Training = 0%

In the next simulation the rate of decay of the relative age effect (r) was increased to 30% while all other variables remained the same as before. The results are illustrated in figure 8. In tier one, 49% of the individuals were of the older, high ability type; 21% were of the older, low ability type; 18% were of the younger, high ability type; and 12% were of the younger, low ability type. In total, 70% of tier one were older individuals and 30% were from the younger categories. In comparison to figure 7, participation by older players in tier one fell from 85% to 70% of the total number of tier one individuals. Participation by individuals from the younger categories increased from 15% to 30% of the tier one population.

In tier two, 62% of the population were from the older categories; 33% from the high ability type and 29% from the low ability type. 38% of tier two were made up of younger players. 21% of tier two were of the younger, high ability type and 17% were of the younger, low ability type. In comparison to figure 7, participation by older players dropped by 9% of the tier two population whereas participation by younger players increased by 9%.

Of the individuals who had exited by the fifth period, 12% were older, high ability types; 18% were older, low ability types; 23% were younger, high ability types; and 47% were younger low ability types. In comparison to figure two, exit by older players increased by 3% of the total, while exit by younger players decreased by 3%. Interestingly, exit by high ability individuals from both age cohorts increased each by 4% of the total, as compared to figure 6. The number of low ability individuals from both age cohorts actually decreased as a percentage of total exits; each low ability age cohort fell by 1%.

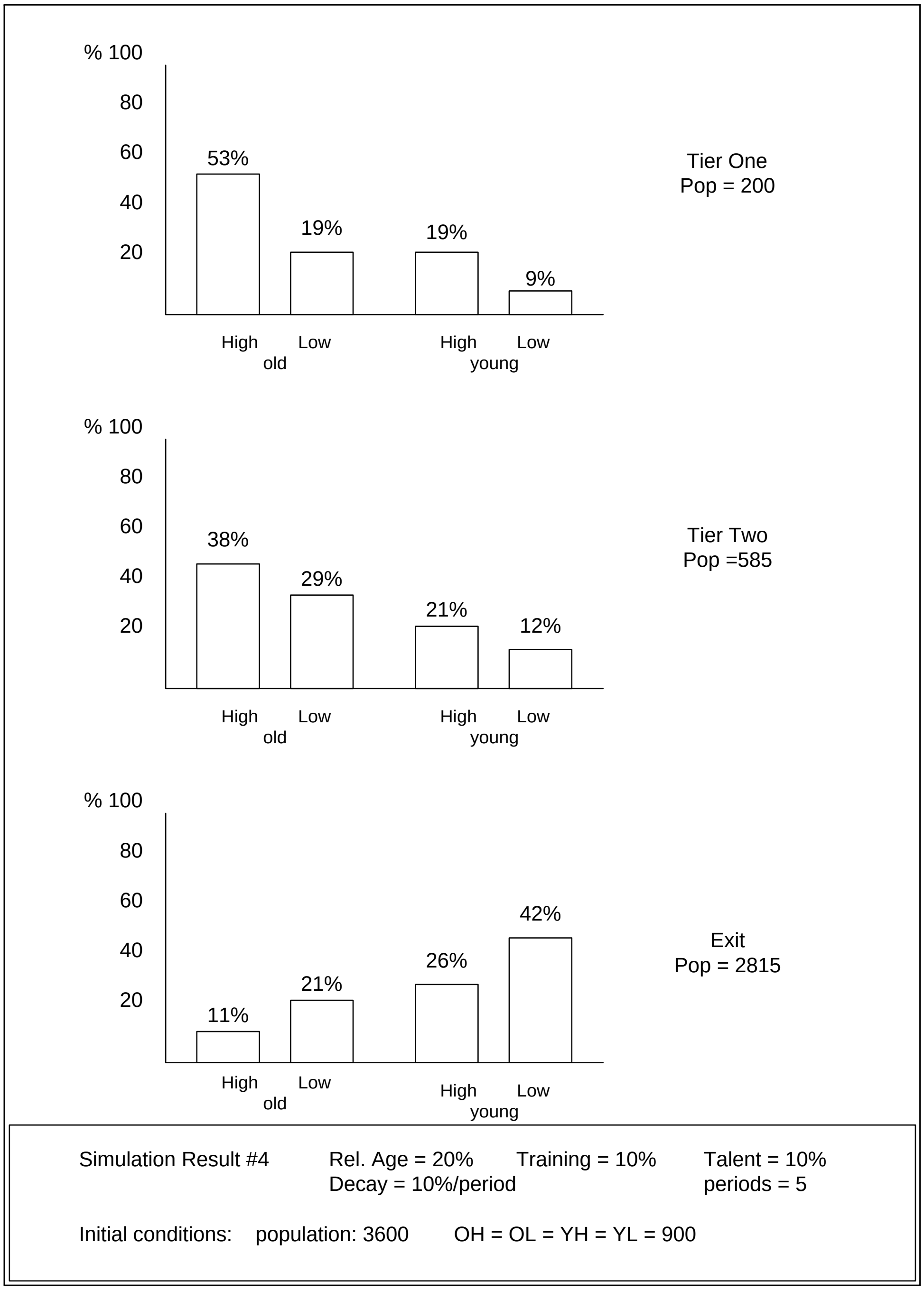
With a 20% initial relative age effect, increasing the rate of decay from 10% to 30% produced the largest change in the mix of tier one participants. The smallest effect occurred on the mix of individuals who exited the system. The number of low ability individuals from both age cohorts who exited the system was practically unchanged, while the number of high ability individuals from both age cohorts that exited increased slightly.

4.2.4. Simulation 4: Relative Age = 20%, Decay = 10%, Training = 10%

The next simulation involved incorporating the training effect. The results are illustrated in figure 9. In this simulation the relative age effect was set at 10%, the rate of decay was set at 10%, and the training effect (s_t) was set at 10%. A 10% training effect means that any individual who was in tier one in any given period will have a 10% better chance of moving to tier one in the next period (ceteris paribus). In this case training effects are not cumulative, therefore, only the previous period matters.

After five periods, the tier one population contained 53% older, high ability types, 19% older, low ability types. 19% younger, high ability types, and 9% younger, low ability types. In total,72% of tier one was made up by individuals from the older age cohort and only 28 % from the younger age cohort. Tier two

FIG. 9 Simulation #4



contained 38% individuals who were of the older, high ability type, 29% older, low ability type, 21% younger, high ability type, and 12% younger, low ability type. In total, tier two contained 67% older individuals and 33% younger individuals.

4.2.5. Simulation 5: Relative Age = 10%, Decay = 10%, Training = 30%

The training effect was then increased to 30% while all the variables were set to the same values as in figure 9. The results are illustrated in figure 10. In tier one, 55% of the population were older, high ability individuals; 26% were older, low ability individuals; 16% were younger, high ability individuals; and 3% were younger, low ability individuals. In all, 81% of tier one were older players and 19% were younger players. Increasing the training effect from 10% to 30% had a small effect on the percentage of older, high ability individuals (53% to 55%) but the percentage of older, low ability individuals did increase from 19% to 26% of tier one. The increased training effect lowered the younger, high ability individuals from 19% to 16% and lowered the younger, low ability individuals from 9% to 3% of tier one.

In tier two, 30% of the population were older, high ability individuals; 29% were older, low ability individuals; 27% were younger, high ability individuals; and 14% were younger, low ability individuals. In comparison to figure 9, the percentage of older, high ability individuals decreased while the other three categories all had percentage increases in participation. In total, 59% of tier two were older individuals and 41% were younger individuals. Interestingly, the distribution of both types of older players and younger, high ability players was approximately uniform (around 30%), and only the younger, low ability players were significantly different, making up only 14% of the tier two population.

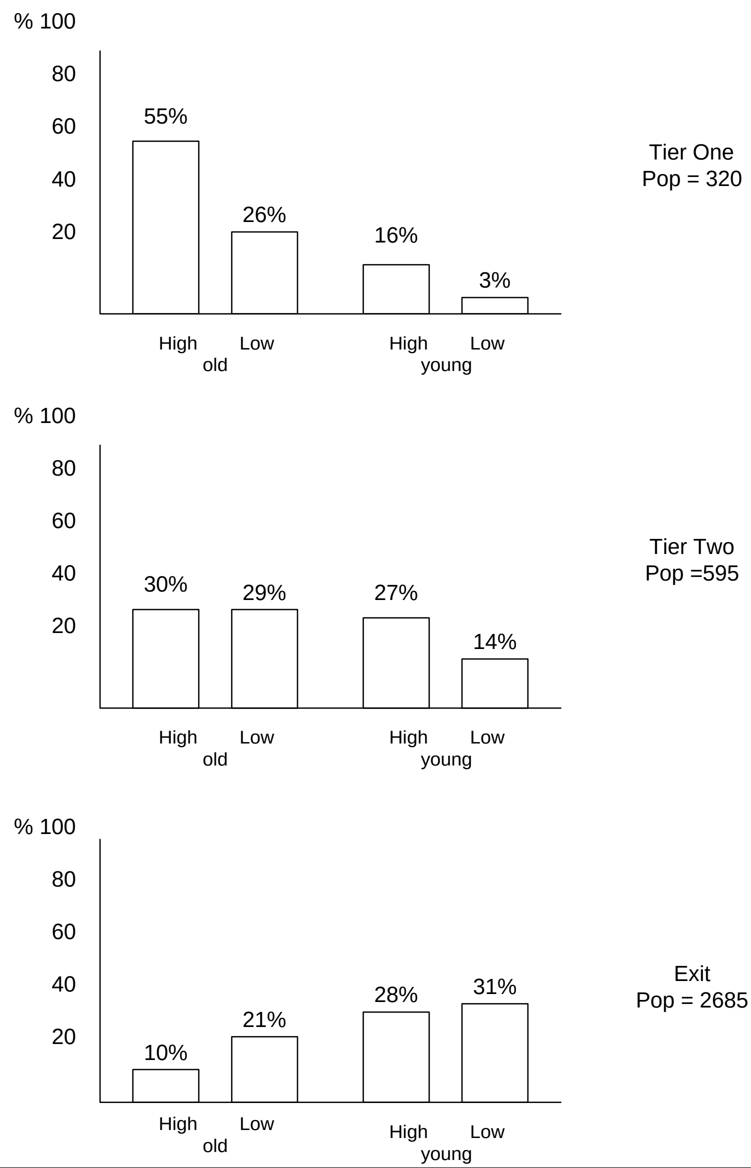
In the exit category, 10% were older, high ability types, 21% were older, low ability types, 28% were younger, high ability types, and 31% were younger, low ability types. Younger players made up 59% of the total number who exited after five periods and 31% were older players.

4.3. General Findings from the Simulations

An extensive number of simulations were run, a sample of which has been presented above. Different functional forms for the probability functions were experimented with, including quadratic and linear models. In general the results were invariant to choice of functional form. Some results were quite obvious and fully expected. First, the smaller the difference in high and low talent the more pronounced was the relative age effects in the final period distributions across tiers. Second, the training effect magnified the relative age effect in tier one. However, when the training effect was relatively large (see figure five) distortions due to relative age in tier two and the exit category were less pronounced.

One interesting result from the simulations was the effect of the decay parameter, r. Even when the rate of decay per period of the relative age was quite large ($r \geq$

FIG. 10 Simulation #5



Simulation Result #5 Rel. Age = 10% Training = 30% Talent = 10%
Decay = 10%/period periods = 5

Initial conditions: population: 3600 OH = OL = YH = YL = 900

30%) the final period distributions in all three categories showed a strong relative age effect. This suggests that it is the magnitude of the relative age effect found in the first period that influences the final distribution rather than the persistence of relative age effects over time.

There were two significant findings that resulted from the model. The first was that even if the relative age effect was small but persisted for several periods (small r value) there would still be the distributions in the final period that matched those found in the data cited. The second result was that the relative age effect could be small and decay rapidly yet the existence of training would produce the same distribution as found when relative age effect was strong. While better data is required for further refining of the relative age and training parameters, one immediate conclusion at this stage is that child development is very sensitive to differentials in training in the early years.

Up to this point the focus of this paper has been on the relative age effect as found in sports; specifically minor hockey. Similar results have also been found in the education system. Research has found a strong correlation between biological maturity and cognitive development. This suggests that the results found in the analysis of sports may also be applied to the education system. The next section presents a discussion of the findings of relative age research in educational performance and learning disabilities.

5. RELATIVE AGE EFFECTS IN EDUCATION

In 1934 Elizabeth Bigelow, a grade one teacher in Summit, New Jersey, published a study which compared the performance of her students to their month of birth[12]. Her basic findings are consistent with the relative age hypothesis. Besides the quantitative results in her study, Ms. Bigelow assessed psychological and sociological problems observed in the classroom that was attributed to the relative age effect. Her conclusions suggested a high social cost to allowing chronologically younger children into a school system too early. Ms. Bigelow's work is probably the first documented study that identified the relative age effect in education. The more recent studies of the relative age effect in education cited in the introduction of this paper show results consistent with the findings of Ms. Bigelow.

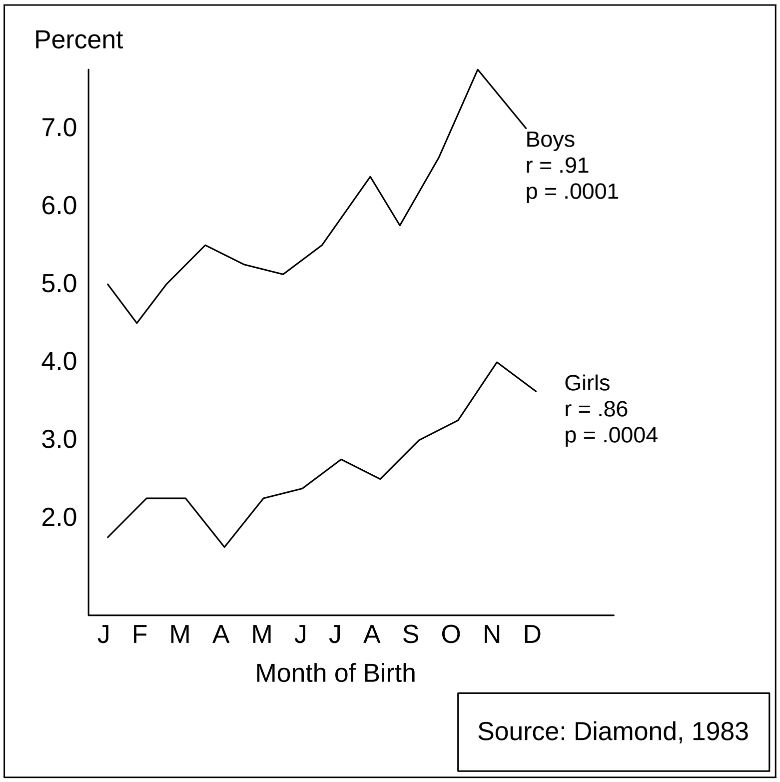
The previous section introduced stylized facts about the relative age effect found in organized minor hockey. Here we turn our attention to the education system. In most countries the education system closely parallels the Canadian minor hockey system in structure. The school system is not as rigid as minor hockey; some children can be held back and others can be moved ahead by a year. But, by in large, most children found in a given grade are born within the same calendar year. Therefore we would expect to observe the relative age effect in performance among the early grades of school.

[12] Bigelow, Elizabeth B., "School Progress of Under- Age Children," Elementary *School Journal* (1934) 25, 186-192

Birthdates	Number of Boys	Number of Girls	Percentage Totals		
January					
February				4%	
March	0	2	4%		
April	3	1	8%		28%
May	1	0	2%	24%	
June	5	2	14%		
July	3	2	10%		
August	3	2	10%	31%	
September	4	2	12%		72%
October	3	2	10%		
November	6	4	20%	41%	
December	4	2	12%		
	32	19			100%

FIG. 11 Summary of Grade one retentions (Alberta School District, 1985)

FIG. 12 Percentage of children born each month classified as learning disabled



5.1. Relative Age in Grade One

Much of the focus of relative age research in education has been on the impact on students when they first enter the education system, typically grade one. Figure 11 presents a summary of retentions in grade one by month of birth. We see that 72% of children who are held back in grade one are born in the second half of the calendar year, and 41% born in the last three months. Similar studies have shown that the results found in figure 11 are consistent across school districts[13]. Further, findings from an ongoing study by Barnsley, Allen, and Thompson[14] show that retentions at the grade three, six and nine levels display similar results as those found in figure 11. They found that 10% of grade nines retained were born in the first quarter of the calendar year, while 40% were born in the last quarter of the calendar year. Their findings suggest that the relative age has a long run, or permanent, effect in academic performance.

[13]See Barnsley, R.H. "Children Starting School: Readiness vs. Relative Age" *Educational Leadership*, (1986) 43, 91-92

[14]Barnsley, R.H., Allen, J., and Thompson, A.H. "School Achievement, Grade Retention and 'The Relative Age Effect'," *ongoing research.(from personal communication with authors)*

5.2. Relative Age and Special Education Placements

Research has shown that the age of entry to school is related to the incidence within some classifications of exceptional children. Diamond (1983) and Maddux (1980) both found that children born in later in the calendar year were over represented in programs for children with learning disabilities. Figure 12 shows the percentage of children born in each month classified as learning disabled. Further, Maddux has shown that children who possess a relative age advantage are over represented as a group in programs for gifted children.

5.3. Relative Age and Academic Achievement

A common feature of most education systems is the periodic administering of achievement tests. In Canada such tests are administered at the grade 3, grade 6, and grade 9 levels. In Great Britain, there are two national tests, known as the *O-levels (age 13)* and *A-levels (age 17)*. Historically, these types of tests have lead to explicit and/or implicit streaming of the young. Figure 13 shows a summary of results of the nationally administered *Canadian Achievement Test (CAT)* for grades three, six and nine from the Lethbridge school district No 51. What is interesting about the data is that the persistence of the relative age effect through the first six years of education. Further, when comparing the 80% and 90% percentile in grade three, the relative age effect appears strongest when moving to the "tails" of the distribution; where scores are used to identify both "learning gifted" and "learning disabled" students.

6. CONCLUSION

A large number of researchers in the fields of education, psychology, and child development have extensively documented the relative age effect in sports and education. This effect has been linked to issues of grade retention and systematic biases in identifying children who have been labeled as *learning disabled*. Two hypotheses have been proposed regarding the relative age effect. The first hypothesis has been the training effect, where children are streamed into different classifications for the purposes of advanced training and development. With the existence of the relative age effect there will be a bias towards older children. Though the relative age effect is considered transitory, the influence of differential training will produce long run effects on the distribution of participants in any form of enriched or elite program.

The second hypothesis driving relative age effect is what is termed as the discouragement effect, which refers to children who are born late in the year being identified as learning disabled or poor performers. Therefore, whenever children later born in the year score poorly in tests or perform poorly in program and sports, they wind up with a stigma that they are in some way untalented, or that they lack the ability or skills. Both of these hypotheses are what drives the permanent nature of the relative age effect, so that long past the point where -biologically-relative age becomes irrelevant. Adults will wind up permanently categorized as either overachievers and high talented, while others will be low ability and underachievers. This gives rise to a classic type-1/type-2 measurement problem. Some

RELATIVE AGE AND ACADEMIC ACHIEVEMENT ALBERTA SCHOOL DISTRICT, 1985					
	Winter	Summer	Autumn	N	χ^2
GRADE 3					
Total	34.5	34.7	30.6	480	
80th %tile +					
Math CAT	56.1	31.6	12.3	57	12.7 (p<.005)
ReadingCAT	52.5	30.5	16.9	59	9.9 (p<.010)
90th %tile +					
Math CAT	71.4	21.4	7.1	14	8.9 (p<.025)
ReadingCAT	75.0	18.8	6.3	16	12.2 (p<.005)
GRADE 6					
Total	32.7	34.3	32.9	434	
80th %tile +					
Math CAT	41.2	34.1	24.7	86	3.1 (p<.250)
ReadingCAT	39.6	33.1	27.3	139	3.2 (p<..250)
GRADE 9					
Total	31.9	33.9	34.1	504	
80th %tile +					
Math CAT	36.3	32.9	30.8	88	0.8 (n.s.)
Reading CAT	32.6	31.1	36.4	132	0.5 (n.s.)
Math& Read	34.2	32.2	33.6	77	0.2 (n.s.)

FIG. 13 Relative age effect in Canadian Achievement Tests (CATs)

talented but relatively young children will be overlooked in the streaming process while relatively older children may be mistakenly labeled as gifted when they are simply average. Both of these errors imply a social cost. There is the cost to the individuals from either the loss due to missed opportunities or the stress and frustration of being expected to perform at a level greater than their abilities will allow.

Of the studies cited, none have been able to analyze the relative age effect in any systematic way. What this extensive body of research has accomplished is to document a widespread nature of this phenomenon. However without any kind of structural model, no true analysis can be carried out, and the ability to derive any meaningful policy recommendations are limited due to the inability to identify the magnitude or the duration of the Relative Age Effect. As an example, on several occasions provincial governments have explored the possibility of going into a dual entry model; where students would begin school in six month intervals rather than in an annual model. The most recent version would be the British Columbia provincial government's *Project 2000* initiative, which was eventually put on hold due to its potential. Given that the education budget is second only to health at the provincial level, the implications of the costs of this initiative would be immense. While there was a recognition of the relative age effect, analysts where not able to offer sufficient evidence to guide the scale or the duration of the project.

The question to be asked is whether or not this program is needed for the entire duration for a student's academic career, or if it could be dealt with within the first 3 or 6 years of education. So the purpose of the structural model developed in this paper would allow us to address those kind of questions directly. By using the structural model to extract different influences of relative age, natural ability and training, we're capable of comparative static exercises that may address some of the policy issues this problem raises.

The model allows us to identify when the relative age effect will cease to distort the screening mechanisms. We can also determine at what point differential training effects compound the problem. Through the use of this kind of model, we would be able to comment whether or not there should be prohibition on enriched classes in first few years of schooling. Given the immense cost of the dual entry system, such as *Project 2000*, it would be prudent to be able to identify for how many years such a structure should be put in place. One of the results of the simulations was that even if the Relative Age effect decayed rapidly, the existence of any significant training during that period would still produce the kinds of long run distributions currently observed in the kind of studies now done in both sports and education.

One of the features of the structural model is that with the appropriate data it we have the ability to extract the natural ability parameters from the other influences. The data gives us a set of observable variables: we know who has received differential training and the relative age of members of cohorts. What can not be observed directly is - for any given group-, is the number who possess high ability or talent. By using maximum likelihood techniques in structural model, we're able to estimate and isolate natural ability from the influences of relative age and training. In essence, we get to observe and unobservable.

There were two significant findings that resulted from the model. The first was that even if the relative age effect was small but persisted for several years, there would still be the distributions in the final period that matched those we have observed. The second result was that the relative age effect could be small and decay rapidly yet the existence of training would produce the same distribution as found when relative age effect was strong. While more research and better data is required for further refining of the relative age model built here, one immediate conclusion at this stage is that child development is very sensitive to differentials in training in the early years. Therefore deterring the use of explicit or implicit enriched programs in school systems during the formative years would go a long way to reducing the distortions that have been associated with relative age effect.

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8. APPENDIX I: CALIBRATION OF THE MODEL

This section presents a brief description of the calibration of the model. The simulation process described earlier was designed to replicate a typical minor hockey system. Therefore the model was calibrated to fit a representative sample. The data was taken from the Edmonton, Alberta minor hockey association for the years 1983-1984[15]. The data covered five levels of organized minor hockey: (i) 9 and 10 year olds (Mite); (ii) 11 and 12 year olds (Pee Wee); (iii) 13 and 14 year olds (Bantum); (iv) 15 and 16 year olds (Midget); (v) 17 and 18 year olds (Juvenile). The data is summarized in 14, which shows the number of players in each level based on quarter of birth. For the levels Mite through Midget, the number of tier one players by quarter of birth is also given. The final level (Juvenile) is a tier one league. The relative size of the population at each level implicitly determined the number of players that exited the system.

The model was calibrated to approximate the aggregate date at each level as determined by a five period Markov process. For purposes of the simulation the first two quarters (Q-1 and Q-2) were aggregated into Old and the last two quarters were aggregated into Young (Q-3 and Q4). It was further assumed that each group could be further divided into two equal groups: high and low natural ability. In the model there are the three binary variables whose coefficients need to be determined (Talent, Age, and Training). Furthermore, there is a fourth variable; the rate of decay of the relative age effect.

The calibration process was carried out in two steps. The first step was a maximum likelihood estimate of the relative age and talent parameters. Of the four variables in the, only relative age is observable. Since the data is aggregate, the training effect can not be directly identified. This would require historical data on individual players- which was not available. However, since there is no training effect in the first period, only the age and talent parameters would be relevant. The results of a maximum likelihood estimate would allow for prediction of the number of high and low ability players in each cohort that moved to tier one, tier two, or exited in the first period transition. The derivation of the first period likelihood function is given below.

Using the results of the maximum likelihood estimates, we then carried out stage two of the calibration process. First we assumed that the coefficient on talent would remain constant across all periods of the Markov process. Then the age and training effects were initially set equal to zero for periods 2 though 5. Then, by imposing restrictions on the aggregate populations found in tier one and tier two at each level, estimates for α_0^t and β_0^t $(t = 2..5)$ using a linear programing algorithm. The associated probabilities in each period's transition matrix produced the results found in simulation one of the chapter.

Deriving The First Period Likelihood Function

Initially, all individuals of each of the j cohorts are at the Mite level ($j = old, young$). Let N_j denote the number of individuals in cohort j. Of the N_j individuals, it is assumed that, initially, half are of high ability and half are of low

[15] Source Barnsley and Thompson (1988)

Level	Tier	Old	Young	total	Total all tiers	Exit
1 Mite	n/a	1132	1078	2210	2210	
8 & under		51%	49%	100%		
2 Pup	Tier 2	760	812	1572		
9-10 yr		34%	37%	71%		
	Tier 1	216	106	322	1894	316
		10%	5%	15%	86%	
3 Peewee	Tier 2	602	623	1225		
11-12 yr		32%	33%	65%		
	Tier 1	213	116	329	1554	340
		11%	6%	17%	82%	
4 Bantum	Tier 2	475	488	963		
13-14 yr		31%	31%	62%		
	Tier 1	148	73	221	1184	370
		10%	5%	14%	76%	
5 Midget	Tier 2	314	297	611		
15-16 yr		27%	25%	52%		
	Tier 1	81	44	125	736	448
		7%	4%	11%	62%	

FIG. 14 Minor Hockey Data (CAHA, Edmonton Alberta 1983-84)

ability. From the N_j individuals, a certain number are selected for tier one, (state A). Let m_j denote the number of players from cohort j at the Peewee Pup level who are selected to state A Peewee in the next period. The progression tree from Peewee Pup to Peewee is given in figure one.

Therefore, for any one player selected at random, the probability that he was of either type was 0.5. If the player was a high ability type, then he would be selected for A with probability $P_{Hj}(A)$. If he was a low ability type then he would be selected to A with probability $P_{Lj}(A)$. Given an individual is selected at random from cohort j, the probability that that individual would move to state A hockey in the next period would be

$$P(A|O) = 0.5P_A^{OH} + 0.5P_A^{OL} \tag{4}$$

$$P(A|Y) = 0.5P_A^{YH} + 0.5P_A^{YL} \tag{5}$$

Therefore, the likelihood function associated with exactly m_j individuals form a population of N_j being selected for state A hockey is

$$\begin{aligned} L_{Old} &= \textstyle\sum_{i=0}^{m_o} \binom{m_o}{i} (P(A|O))^i \binom{N_o}{m_o - i} (1 - P(A|O))^{(N_o + i - m_o)} \\ &= \textstyle\sum_{i=0}^{m_o} \binom{m_o}{i} (P(A|O))^i \binom{N_o}{m_o - i} (P(B|O) \\ &\qquad + P(E|O))^{(N_o + i - m_O)} \end{aligned} \tag{6}$$

$$\begin{aligned} L_{young} = & \textstyle\sum_{i=0}^{m_y} \binom{m_y}{i} (P(A|y))^i \binom{N_y}{m_y - i} (1 - P(A|y))^{(N_y + i - m_y)} \\ &= \textstyle\sum_{i=0}^{m_{yg}} \binom{m_y}{i} (P(A|y))^i \binom{N_y}{m_y - i} (P(B|y) \\ &\qquad + P(E|y))^{(N_y + i - m_y)} \end{aligned} \tag{7}$$

To estimate the relative age and talent parameters across all four cohorts simultaneously, the likelihood function to be estimated, denoted LL is simply the product of the likelihood functions for each of the j cohorts, or

$$LL = (L_{old})(L_{young}) \tag{8}$$

In the first stage the parameters to be estimated are α_0, α_1, α_2, $\beta_{0,}$ β_1, and β_2.