

Find f_{xy} for

$$f(x, y) = 8x^4y^3 - 5x^5y^6$$

First find f_x

$$\begin{aligned} f_x &= 8(4x^3)y^3 - 5(5x^4)y^6 \\ f_x &= 32x^3y^3 - 25x^4y^6 \end{aligned}$$

Now find f_{xy} which is $\partial(f_x)/\partial y$

$$\begin{aligned} f_{xy} &= \frac{\partial f_x}{\partial y} = 32x^3(3y^2) - 25x^4(6y^5) \\ f_{xy} &= 96x^3y^2 - 150x^4y^5 \end{aligned}$$

Now, let's do the derivative in reverse order. Let's find f_{yx}

$$f(x, y) = 8x^4y^3 - 5x^5y^6$$

First find f_y

$$\begin{aligned} f_y &= 8x^4(3y^2) - 5x^5(6y^5) \\ f_y &= 24x^4y^2 - 30x^5y^5 \end{aligned}$$

Now find f_{yx} which is $\partial(f_y)/\partial x$

$$\begin{aligned} f_{yx} &= 24(4x^3)y^2 - 30(5x^4)y^5 \\ f_{yx} &= 96x^3y^2 - 150x^4y^5 \end{aligned}$$

Notice that the answer is the same both ways. This is known as Young's Theorem, which says that, for cross-partial derivatives, the order of differentiation does not matter, i.e.

$$f_{xy} = f_{yx}$$